A comparison between CQM and Bag model calculations for the Sivers and Boer-Mulders functions

Workshop in TMDs

EINN09, Milos

Aurore Courtoy

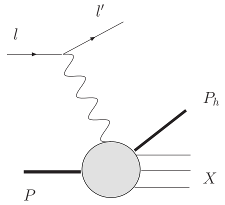
Universidad de Valencia

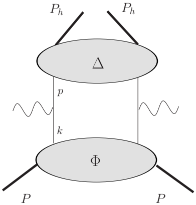
with S. Scopetta (Perugia) and V. Vento (Valencia)

29/09/09

Outlines

Prototype Process: Semi-Inclusive Deep Inelastic Scattering





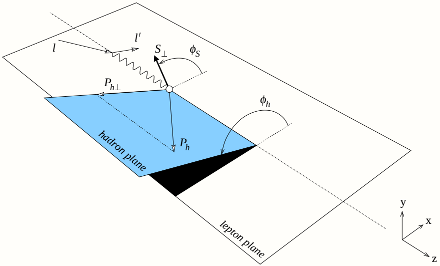
SIDIS: $l(l) + N(P) \rightarrow l(l') + h(P_h) + X$

Factorization

$$W^{\mu\nu} \propto \sum_q e_q^2 \int d^4p d^4k \, \Phi(k, P, S)\gamma^\mu \Delta(p, P_h)\gamma^\nu$$

- $\Phi(k, P, S) \Rightarrow$ Parton Distribution Functions
- $\Delta(p, P_h) \Rightarrow$ Fragmentation Functions
- Nonperturbative Objects

Some Asymmetries in SIDIS



- ϕ_h = angle between leptonic and hadronic planes
- ϕ_S = angle between leptonic plane and transverse spin of the target
- Trento Convention [PRD70, 117504]

Azimuthal Asymmetries for unpolarized target in SIDIS

$$e.g., \qquad A(\phi_h) \Rightarrow \langle \cos \phi_h \rangle, \langle \cos 2\phi_h \rangle$$

Single-Spin Asymmetries for transverse target polarization in SIDIS

$$e.g., \qquad A(\phi_h, \phi_S) \Rightarrow \langle \sin(\phi_h - \phi_S) \rangle, \langle \sin(\phi_h + \phi_S) \rangle$$

Transverse Momentum Dependent PDF

Non-perturbative effects of the **intrinsic transverse momentum $\vec{k}_\perp$ of the quarks** inside the nucleon may induce significant hadron azimuthal asymmetries.

[Cahn; Mulders & Tangermans, . . .]

- Relaxing Time-reversal Invariance $\Rightarrow$ *naive* T**-odd** functions,

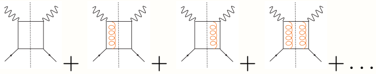
  e.g. **Sivers & Boer-Mulders functions**

  [Sivers, PRD41];[Boer & Mulders PRD57.]

- Existence of **Final State Interactions** at leading-order

  [Brodsky, Hwang & Schmidt, PLB 530];[Belitsky, Ji &Yuan NPB 656.]

- The gauge link:



0th order, No gauge link $\longrightarrow$ T-odd fct $= 0$

Existence of leading-twist FSI $\longrightarrow$ T-odd fct $\neq 0$

Models

The Sivers function $f_{1T}^{\perp Q}(x, k_T)$

⇒ Distribution of **unpolarized quarks** inside a **transversely polarized proton**

and

The Boer-Mulders functions $h_1^{\perp Q}(x, k_T)$

⇒ Distribution of **transversely polarized quarks** inside a **unpolarized proton**

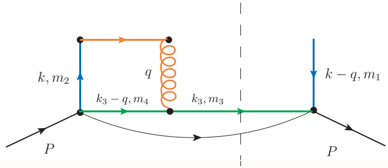
- non-perturbative quantities ⟶ not calculable in QCD
- we use models for the proton ⟶ not an exact calculation
- goal ⟶ insights into microscopic mechanisms
- HERE: formalisms for
  - **MIT bag model** e.g. [Jaffe, PRD11]
  - **Constituent Quark Models (CQM)** e.g. [de Rújula, Georgi & Glashow, PRD12]

Formalisms for the T-odd functions

I. Constituent Quark Model $\longrightarrow$ 3-body

Recipe

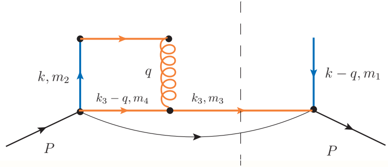
- go to a helicity basis [Sivers],
- expand the free quark fields
- properly insert complete sets of free states



- identify the intrinsic proton w.f. $\Psi_{r\,S_z}$
- we are left with the interaction term

$$V(\vec{k}_1, \vec{k}_3, \vec{q}) = \frac{1}{q^2}\bar{u}_{m_1}(\vec{k}-\vec{q})\Gamma u_{m_2}(\vec{k})\bar{u}_{m_3}(\vec{k}_3)\gamma^+ u_{m_4}(\vec{k}_3-\vec{q})$$

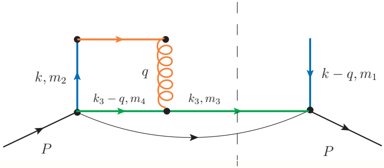
$\longrightarrow$ to be reduced NR (in a CQM fashion)



[A.C., F. Fratini, S. Scopetta and V. Vento, PRD78]

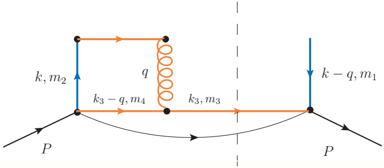
Formalisms for the T-odd functions

II. MIT Bag Model $\longrightarrow$ **1-body**



Recipe

- go to a helicity basis [Sivers],
- expand into the bag quark w.f.
- properly insert complete sets of free states



- we are left with the interaction term
  $V(\vec{k}_1, \vec{k}_3, \vec{q})$
  $= \frac{1}{q^2} \varphi^\dagger_{m_1}(\vec{k}-\vec{q})\gamma^0\Gamma\varphi_{m_2}(\vec{k})\varphi^\dagger_{m_3}(\vec{k_3})\gamma^0\gamma^+\varphi_{m_4}(\vec{k_3}-\vec{q})$

[F. Yuan, PLB575]

The interaction

I. Constituent Quark Model

NR reduction of the interaction - up to $O\left(\frac{k^2}{m^2}\right)$-

Use of free spinors ⟶

$$u_m(\vec{k}) \propto \begin{pmatrix} \chi_m \\ \frac{\vec{\sigma}\cdot\vec{k}}{k^0+m}\chi_m \end{pmatrix}$$

$f_{1T}^{\perp\mathcal{Q}}, h_1^{\perp\mathcal{Q}} \neq 0$ comes from Interference of the lower and upper components in the four-spinors of the **free quark states**

II. MIT Bag Model

Bag wave function ⟶

$$\varphi_m(\vec{k}) \propto \begin{pmatrix} t_0(|\vec{k}|)\chi_m \\ \vec{\sigma}\cdot\hat{k}\, t_1(|\vec{k}|)\,\chi_m \end{pmatrix}$$

$f_{1T}^{\perp\mathcal{Q}}, h_1^{\perp\mathcal{Q}} \neq 0$ comes from the Interference of the lower and upper components in the **bag w.f.**

The interaction

I. Constituent Quark Model

NR reduction of the interaction - up to $O\left(\frac{k^2}{m^2}\right)$-

Use of free spinors ⟶

$$u_m(\vec{k}) \propto \begin{pmatrix} \chi_m \\ \frac{\vec{\sigma}\cdot\vec{k}}{k^0+m}\chi_m \end{pmatrix}$$

$f_{1T}^{\perp\mathcal{Q}}, h_1^{\perp\mathcal{Q}} \neq 0$ comes from Interference of the lower and upper components in the four-spinors of the **free quark states**

The interaction is to be calculated between proton states in a CQM ⇒ e.g., Harmonic Oscillator $|N\rangle = a|^2S_{1/2}\rangle_S$

⇒ **SU(6) symmetry for the proton**

II. MIT Bag Model

Bag wave function ⟶

$$\varphi_m(\vec{k}) \propto \begin{pmatrix} t_0(|\vec{k}|)\chi_m \\ \vec{\sigma}\cdot\hat{k}\, t_1(|\vec{k}|)\,\chi_m \end{pmatrix}$$

$f_{1T}^{\perp\mathcal{Q}}, h_1^{\perp\mathcal{Q}} \neq 0$ comes from the Interference of the lower and upper components in the **bag w.f.**

The interaction is to be calculated between proton states, we choose ⇒ **SU(6) symmetry for the proton**

Properties of the Sivers function

- **Experiment**
  - Evidence for **non-zero** Sivers function at HERMES [2003]
  - Sivers Asymmetry **statistically compatible with zero** within present statistical error at COMPASS [from *Transversity 2008*]
  - Future: CLAS@12GeV?
- **Extraction from data**
  - W. Vogelsang and F. Yuan, PRD 72, 054028 (2005)
  - M. Anselmino et al., Eur.Phys.J.A39:89-100,2009
  - J.C. Collins et al., PRD 73, 014021 (2006)
- **Theory: Properties of the Sivers function**
  - From first principles: Burkardt Sum Rule (PRD 69 (2004) 091501)

$$\sum_{\mathcal{Q}=u,d} \langle k_x^{\mathcal{Q}} \rangle = \sum_{\mathcal{Q}=u,d} -\int_0^1 dx \int d\vec{k}_T \frac{k_x^2}{M} f_{1T}^{\perp\,\mathcal{Q}}(x, k_T) = 0$$

  - Hypothetical relation with the E GPD $\longrightarrow$ distribution for u is negative, $\longrightarrow$ distribution for d is positive.
  - Quarks Orbital Angular Momentum (Burkardt,...)

Model Calculations with SU(6) proton WF and perturbative OGE

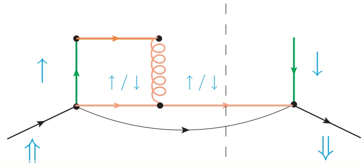
- **NR Constituent Quark Model** $\longrightarrow$ **3-body**

  [A.C., Fratini, Scopetta and Vento, PRD 78 (2008).]

  - No proportionality u and d distribution
  - *Small* Violation of the Burkardt SR

  $$\frac{\langle k_x^u \rangle + \langle k_x^d \rangle}{\langle k_x^u \rangle - \langle k_x^d \rangle} \simeq \mathbf{0.02}$$

  - **Both** active OR non-active quark helicity-flip



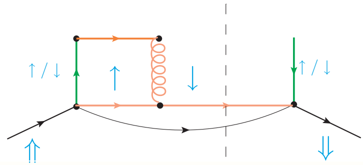
- **MIT Bag Model** $\longrightarrow$ **1-body**

  [Yuan, PLB 575 (2003)]

  - Proportionality u and d distribution
  - *Large* Violation of the Burkardt SR

  $$\frac{\langle k_x^u \rangle + \langle k_x^d \rangle}{\langle k_x^u \rangle - \langle k_x^d \rangle} \simeq \mathbf{0.60}$$

  - **Only** active quark helicity-flip



Model Calculations with SU(6) proton WF and perturbative OGE

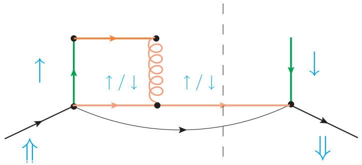
- **NR Constituent Quark Model** $\longrightarrow$ **3-body**

  [A.C., Fratini, Scopetta and Vento, PRD 78 (2008).]

  - No proportionality u and d distribution
  - *Small* Violation of the Burkardt SR

  $$\frac{\langle k_x^u \rangle + \langle k_x^d \rangle}{\langle k_x^u \rangle - \langle k_x^d \rangle} \simeq 0.02$$

  - **Both** active OR non-active quark helicity-flip



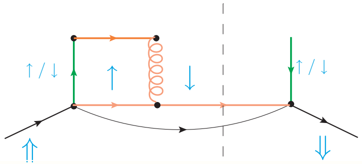
- **MIT Bag Model** $\longrightarrow$ **1-body**

  [Yuan, PLB 575 (2003)]

  - Proportionality u and d distribution
  - *Large* Violation of the Burkardt SR

  $$\frac{\langle k_x^u \rangle + \langle k_x^d \rangle}{\langle k_x^u \rangle - \langle k_x^d \rangle} \simeq 0.60$$

  - **Only** active quark helicity-flip



Why such a difference between those 2 models?

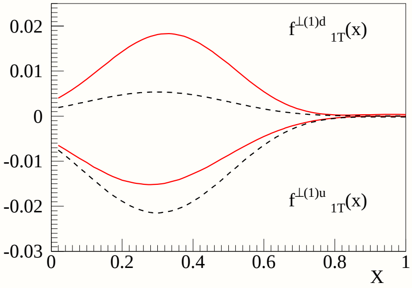
Constituent Quark Model vs. MIT Bag Model

Solution

The "non-active quark" can also flip helicity in the MIT Bag Model!!!

[A. C., Scopetta & Vento, PRD79]

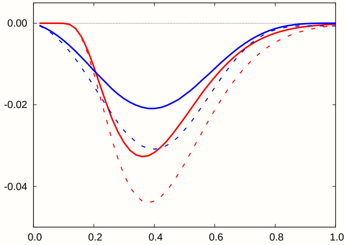
- **Before:** dashed curve
  - Missing term in the first calculation in the bag
  - Burkardt SR violated by 60%
- **After:** plain red curve
  - Helicity-flip of the "non-active quark" taken into account
  - Burkardt SR violated by 5%

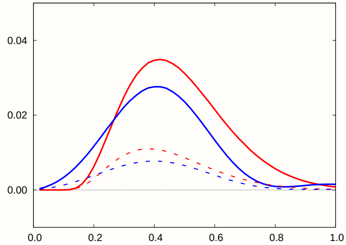


The results in the MIT Bag Model fulfill the theoretical properties of the Sivers function

No proportionality between the *u* and *d*-distributions

Helicity-flip contributions

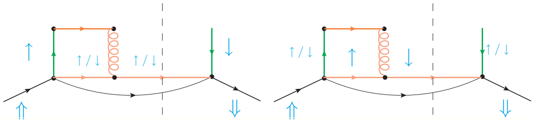




1st moment

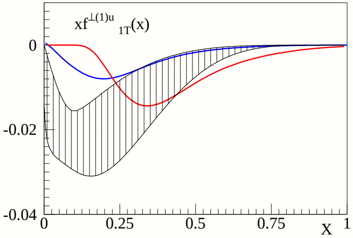
$$f_{1T}^{\perp(1)q}(x) = \int d^2\vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T)$$

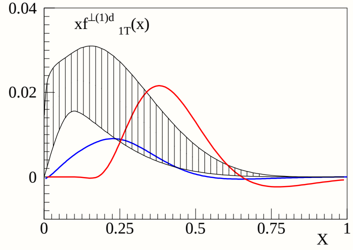
- u upper; d lower panels
- **red curves**: results in a CQM (H.O.)
- **blue curves**: results in the MIT bag model
- **full curves**: full results
- **dashed curves**: results interacting quark flipping helicity



Both Models at the hadronic scale $\mu_o^2 \sim 0.1$ GeV^2

Results in a CQM





1st moment

$$f_{1T}^{\perp(1)q}(x) = \int d^2\vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T) \ .$$

- **full**: results at the hadronic scale $\mu_o^2 \simeq 0.1$ GeV^2
- **shaded area**: 1-σ region of the best fit of the Sivers function extracted from HERMES data, at $Q^2 = 2.5$ GeV^2

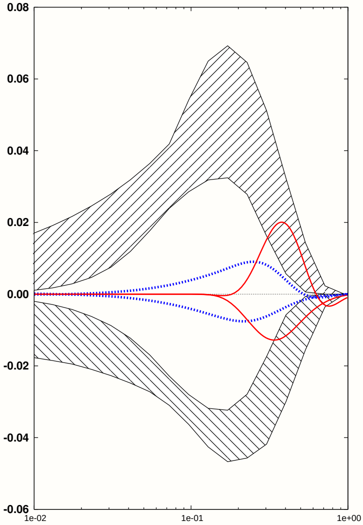
[Collins et al., PRD 73 (2006) 014021]

Model at ~ 0.1 GeV^2 vs. Exp. at 2.5 GeV^2

- Evolution
  - **Blue**: results after NLO-*standard* evolution to $Q^2 = 2.5$ GeV^2
  - Correct evolution missing

The results in the CQM are $\sim$ in agreement with this extraction of the Sivers function

Results in a CQM



1st moment

$$f_{1T}^{\perp(1)q}(x) = \int d^2\vec{k}_T \, \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T) \; .$$

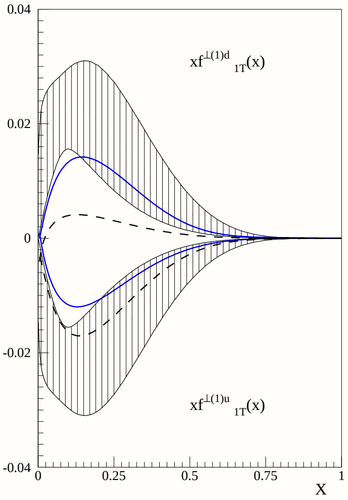
- **full**: results at the hadronic scale $\mu_o^2 \simeq 0.1$ GeV^2
- **Shaded Area:** Sivers function extracted from HERMES and COMPASS data
  [M. Anselmino et al., Eur. Phys. J. A39:89-100 (2009)]

Model at ~ 0.1 GeV^2 vs. Exp. at 2.5 GeV^2

- Evolution
  - **Blue**: results after NLO-*standard* evolution to $Q^2 = 2.5$ GeV^2
  - Correct evolution missing

The results in the CQM are $\sim$ in agreement with this extraction of the Sivers function

Revised Results in the MIT Bag Model



- **Shaded Area:** 1-σ region of the best fit of the Sivers function extracted from HERMES data, at $Q^2 = 2.5$ GeV^2

  [Collins et al., PRD 73 (2006) 014021]

- **Dashed Curve:** 1st result in the MIT Bag model *after NLO-standard evolution*

  [Yuan, PLB 575 (2003)]

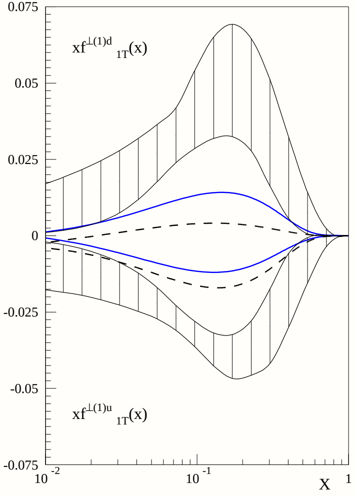
- **Plain blue Curve:** revised result in the MIT Bag model *after NLO-standard evolution*

  [A. C., Scopetta & Vento, PRD79]

The results in the MIT Bag Model are in agreement with this extraction of the Sivers function

. . . up to correct Evolution of the Sivers function

Revised Results in the MIT Bag Model



- **Shaded Area:** Sivers function extracted from HERMES and COMPASS data
  [M. Anselmino et al., Eur. Phys. J. A39:89-100 (2009)]
- **Dashed Curve:** 1st result in the MIT Bag model *after NLO-standard evolution*
  [Yuan, PLB 575 (2003)]
- **Plain blue Curve:** revised result in the MIT Bag model *after NLO-standard evolution*
  [A. C., Scopetta & Vento, arXiv:0811.1191 [hep-ph]]

The results in the MIT Bag Model are now in a better agreement with this extraction of the Sivers function

. . . up to correct Evolution of the Sivers function

Properties of the Boer-Mulders function

- **Experiment**
  - Drell-Yan at FNAL [2007]
  - SIDIS at COMPASS and HERMES [2009]
  - Future: PAX@FAIR ?
  - Difficulty of the analysis

- **Extraction from data**

Drell-Yan [Zhang, Lu, Ma and Schmidt, PRD 77]
SIDIS predictions in [Barone, Prokudin and Ma , PRD78]

- **Theory: Properties of the Boer-Mulders function**
  - Hypothetical relation with the chiral-odd $\bar{E}$ GPD [Burkardt and Hannafious, PLB658]

    $\longrightarrow$ distribution for u is negative,
    $\longrightarrow$ distribution for d is negative.

  - From first principles: Lattice [QCDSF and UKQCD Colls., PRL 98]

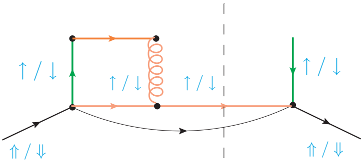
    $\longrightarrow$ moments of chiral-odd GPDs

Model Calculations with SU(6) proton WF and perturbative OGE

- **NR Constituent Quark Model** $\longrightarrow$ **3-body**

  [A.C., Scopetta and Vento, [0909.1404]]

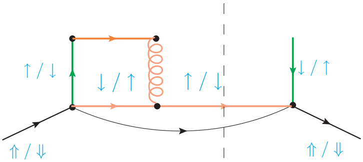
  - No proportionality u and d distribution
  - **Both** no helicity-flip AND active and non-active quark helicity-flip



- **MIT Bag Model** $\longrightarrow$ **1-body**

  [Yuan, PLB 575 (2003)]

  - Proportionality u and d distribution
  - **Only** no helicity-flip

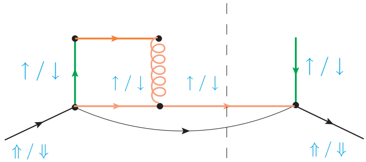


Model Calculations with SU(6) proton WF and perturbative OGE

- **NR Constituent Quark Model** $\longrightarrow$ **3-body**

  [A.C., Scopetta and Vento, [0909.1404]]

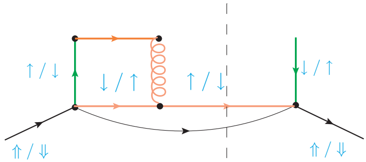
  - No proportionality u and d distribution
  - **Both** no helicity-flip AND active and non-active quark helicity-flip



- **MIT Bag Model** $\longrightarrow$ **1-body**

  [Yuan, PLB 575 (2003)]

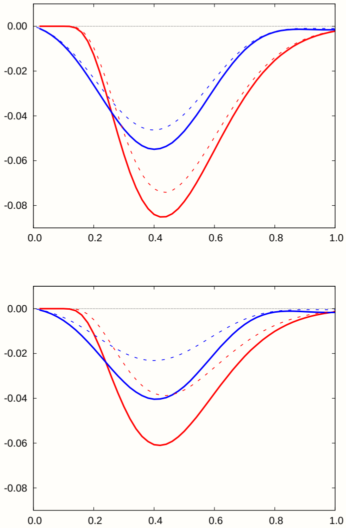
  - Proportionality u and d distribution
  - **Only** no helicity-flip



$\Rightarrow$ Same problem as for the Sivers function

$\Rightarrow$ **We have revised the BM in the bag!!**

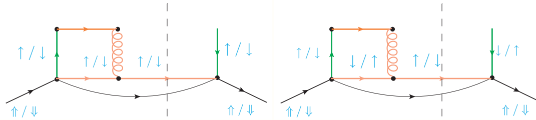
Helicity-flip contributions



1st moment

$$h_1^{\perp(1)q}(x) = \int d^2\vec{k}_T \, \frac{k_T^2}{2M^2} \, h_1^{\perp q}(x, k_T) \ .$$

- *u* upper; *d* lower panels
- **red curves**: results in a CQM (H.O.)
- **blue curves**: results in the MIT bag model
- **full curves**: full results
- **dashed curves**: results non-flipping helicity



Both Models at the hadronic scale $\mu_o^2 \sim 0.1$ GeV^2

Conclusions for the CQM

T-odd functions in CQM:

- Analysis of the Sivers & Boer-Mulders functions in a 3-Body model
  [A.C., Fratini, Scopetta, Vento, PRD78]
- Formalism valid for any CQM
  $\Rightarrow$ **Ingredients**: wave functions and a reduction of the interaction
- Non-Relativistic

Sivers function in the **H.O.**

- Correct relative sign for u and d distributions
- Burkardt sum rule recovered
- Reasonable agreement with data

Boer-Mulders function in the **H.O.**

- Correct relative sign for u and d distributions
- Reasonable agreement with expectations from other evaluations

Conclusions for the MIT Bag Model

T-odd functions in the Bag:

- Analysis of the Sivers & Boer-Mulders functions in a 1-Body model
  [F. Yuan, PLB575]
- Formalism in the bag
  ⇒ **Ingredients**: bag wave functions and SU(6) proton state
- Relativistic

Sivers function in the bag [A.C., Scopetta, Vento,PRD79]

- Correct relative sign for u and d distributions
- Burkardt sum rule recovered
- Reasonable agreement with data

Boer-Mulders function in the bag [A.C., Scopetta, Vento, [0909.1404]]

- Correct relative sign for u and d distributions
- Reasonable agreement with expectations from other evaluations

Conclusions

Conclusions

- **Agreement between models**
  - More *physical* picture for helicity-flip at the quark level
    $\Rightarrow$ either the *active* and *non-active* quark can flip
  - First Principles arguments $\Rightarrow$ Burkardt Sum Rule fulfilled **[Sivers]**

- Much **better agreement** of the MIT Bag calculation with the actual **extractions**

- **By-Product:** confidence on the **Non-Relativistic** expansion !!

- **. . . to the experiments**
  1. Need for Correct **evolution** of TMDs   Stefanis, Cherednikov, Ceccopieri ?
  2. Relation between **GPDs in Impact Parameter Space** and T-odd functions????
  3. **Improvement of the models** after experimental feedback   No boost necessary?

**Do not quench your inspiration and your imagination;
do not become the slave of your model.**

Vincent Van Gogh

Definitions

The Sivers function

Distribution of **unpolarized quarks** inside a **transversely polarized proton**

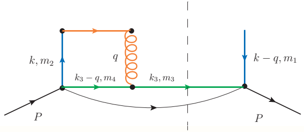
$$\begin{aligned} \mathbf{f}_{1T}^{\perp\mathcal{Q}}(\mathbf{x},\mathbf{k_T}) &= \mathbf{f}^{\mathcal{Q}}_{\mathbf{q/p}\uparrow}(\mathbf{x},\tilde{\mathbf{k}}_\mathbf{T},\mathbf{S}) - \mathbf{f}^{\mathcal{Q}}_{\mathbf{q/p}\downarrow}(\mathbf{x},\tilde{\mathbf{k}}_\mathbf{T},\mathbf{S}) \\ &= -\frac{M}{2k_x}\int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)} \\ &\quad \frac{1}{2}\sum_{S_y=-1,1} S_y \langle P, S_y|\bar{\psi}_{\mathcal{Q}}(0,\xi^-,\vec{\xi}_T)\mathcal{L}^\dagger_{\vec{\xi}_T}(\infty,\xi^-)\gamma^+\mathcal{L}_0(\infty,0)\psi_{\mathcal{Q}}(0,0,0)|P,S_y\rangle \end{aligned}$$

The Boer-Mulders function

Distribution of **transversely polarized quarks** inside a **unpolarized proton**

$$\begin{aligned} \mathbf{h}_1^{\perp\mathcal{Q}}(\mathbf{x},\mathbf{k_T}) &= \mathbf{f}^{\mathcal{Q}}_{\mathbf{q}\uparrow/\mathbf{p}}(\mathbf{x},\tilde{\mathbf{k}}_\mathbf{T},\mathbf{S}) - \mathbf{f}^{\mathcal{Q}}_{\mathbf{q}\downarrow/\mathbf{p}}(\mathbf{x},\tilde{\mathbf{k}}_\mathbf{T},\mathbf{S}) \\ &= -\frac{M}{2k_x}\int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)} \\ &\quad \frac{1}{2}\sum_{S_z=-1,1} \langle P, S_z|\bar{\psi}_{\mathcal{Q}}(0,\xi^-,\vec{\xi}_T)\mathcal{L}^\dagger_{\vec{\xi}_T}(\infty,\xi^-)\gamma^+\gamma^2\gamma_5\mathcal{L}_0(\infty,0)\psi_{\mathcal{Q}}(0,0,0)|P,S_z\rangle \end{aligned}$$

Calculation Details: MIT bag



$$f_{1T}^{\perp\mathcal{Q}}(x, k_\perp) \propto 2\Re\Big\{ \int \frac{d^2 q_\perp}{(2\pi)^5} \frac{i}{q^2} \sum_{\{m\},\beta} C_{\{m\}}^{\mathcal{Q},\beta} \varphi_{m_1}^\dagger(\vec{k} - \vec{q}_\perp)\gamma^0\gamma^+\varphi_{m_2}(\vec{k}) \int \frac{d^3 k_3}{(2\pi)^3} \varphi_{m_3}^\dagger(\vec{k_3})\gamma^0\gamma^+\varphi_{m_4}(\vec{k_3} - \vec{q}_\perp) \Big\}$$

$$\int \frac{d^3 k_3}{(2\pi)^3} \varphi_{m_3}^\dagger(\vec{k_3})\gamma^0\gamma^+\varphi_{m_4}(\vec{k_3} - \vec{q}_\perp) = F(\vec{q}_\perp)\,\delta_{m_3 m_4} + H(\vec{q}_\perp)\,\delta_{m_3,-m_4}$$

With the MIT Bag WF,

$$\varphi_m(\vec{k}) \propto \begin{pmatrix} t_0(|\vec{k}|)\chi_m \\ \vec{\sigma}\cdot\hat{k}\, t_1(|\vec{k}|)\,\chi_m \end{pmatrix} \quad , \qquad t_i(k) = \int_0^1 u^2\, du\, j_i(ukR_0) j_i(u\omega)$$

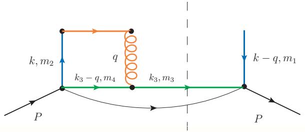
$H(\vec{q}_\perp)$ does not vanish in a basis for the gluon's momentum constrained by the DIS framework,

$\Rightarrow$ *z*-axis is the virtual photon's direction

$\Rightarrow$ operator structure: γ^+

Sivers function in a CQM: Calculation details

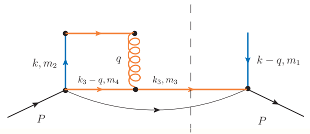
In a helicity basis, to the first non-vanishing order
by expanding the free quark fields
and by properly inserting complete sets of free states



$$
\begin{aligned}
\mathbf{f_{1T}^{\perp Q}(x, k_T)} \quad = \quad & \Im\Big\{ \frac{M}{2k_x} \int \frac{d\xi^- \, d^2\vec{\xi}_T}{(2\pi)^3} e^{-i(x\xi^- P^+ - \vec{\xi}_T \cdot \vec{k}_T)} \langle PrS_z = 1| \\
& \int d\tilde{k}_3 \sum_{m_3} b^{\mathcal{Q}\dagger}_{m_3 i}(\tilde{k}_3) e^{ik_3^+ \xi^- - i\vec{k}_{3T} \cdot \vec{\xi}_T} \bar{u}_{m_3}(\vec{k}_3) \\
& \sum_{l_n, l_1} \int d\tilde{k}_n \int d\tilde{k}_1 |\tilde{k}_1 l_1\rangle |\tilde{k}_n l_n\rangle \langle \tilde{k}_n l_n| \langle \tilde{k}_1 l_1| \\
& (ig) \int_{\xi^-}^{\infty} A_a^+(0, \eta^-, \vec{\xi}_T) d\eta^- \, T^a_{ij} \\
& \sum_{l'_n, l'_1} \int d\tilde{k}'_n \int d\tilde{k}'_1 |\tilde{k}'_1 l'_1\rangle |\tilde{k}'_n l'_n\rangle \langle \tilde{k}'_n l'_n| \langle \tilde{k}'_1 l'_1| \, \gamma^+ \\
& \sum_{m'_3} \int d\tilde{k}'_3 \, b^{\mathcal{Q}}_{m'_3 j}(\tilde{k}'_3) u_{m'_3}(\vec{k}'_3) |PS_z = -1\rangle + \text{h.c.} \Big\} \quad .
\end{aligned}
$$

[A.C., F. Fratini, S. Scopetta and V. Vento, Phys.Rev.D78:034002,2008.]

Sivers function in a CQM: Calculation



Identifying the intrinsic proton wave function:

$$\mathbf{f}_{\mathbf{1T}}^{\perp\mathbf{Q}}(\mathbf{x},\mathbf{k_T}) = \Im\Big\{ ig^2 \frac{M}{2k_x}\int d\tilde{k}_1 d\tilde{k}_3 \frac{d^4q}{(2\pi)^3}\delta(q^+)(2\pi)\delta(q_0)\delta(k_3^+ + q^+ - xP^+)\delta(\vec{k}_{3T}+\vec{q}_T-\vec{k}_T)$$

$$\sum_{\mathcal{F}_1,\{m_i\}\{c_i\}} \Psi^{\dagger}_{r\,S_z=1}\left(\tilde{k}_3\{m_3,i,Q\};\ \tilde{k}_1\{m_1,c_1,\mathcal{F}_1\};\ \tilde{P}-\tilde{k}_3-\tilde{k}_1,l_n\right)\ T^a_{ij}T^a_{c_1c_1'}\frac{1}{q^2}V(\vec{k}_1,\vec{k}_3,\vec{q})$$

$$\Psi_{r\,S_z=-1}\left(\tilde{k}_3+\tilde{q},\{m_3',j,Q\};\ \tilde{k}_1-\tilde{q},\{m_1',c_1',\mathcal{F}_1\};\ \tilde{P}-\tilde{k}_3-\tilde{k}_1,l_n\right)\Big\}$$

with the interaction given by:

$$V(\vec{k}_1,\vec{k}_3,\vec{q}) = \bar{u}_{m_3}(\vec{k}_3)\gamma^+u_{m_3'}(\vec{k}_3+\vec{q})\bar{u}_{m_1}(\vec{k}_1)\gamma^+u_{m_1'}(\vec{k}_1-\vec{q})$$

Next step: reduction of the interaction (in a CQM fashion)

[de Rújula, Georgi, Glashow PRD 12, 147, (1975)]

Calculation details: CQM

NR reduction of the interaction - up to $O\left(\frac{k^2}{m^2}\right)$-

Use of free spinors $\longrightarrow$

$$u_m(\vec{k}) \propto \begin{pmatrix} \chi \\ \frac{\vec{\sigma}\cdot\vec{k}}{k^0+m}\chi \end{pmatrix}$$

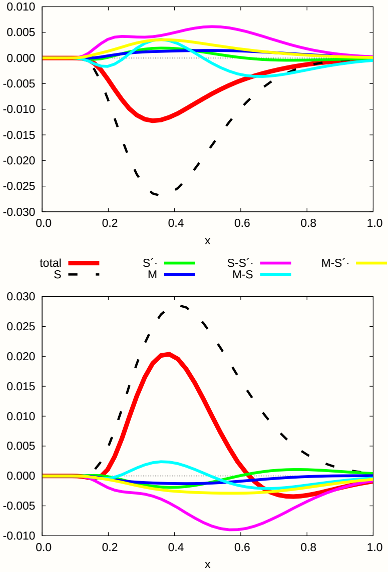
$$\begin{aligned} V(\vec{k}_1, \vec{k}_3, \vec{q}) &= \Bigg\{ -i\frac{(\vec{q}\times\vec{\sigma}_1)_z}{4m^2}\left(1+\frac{k_3^z}{m}+\frac{\vec{q}\cdot\vec{k}_3}{4m^2}\right) + i\frac{(\vec{q}\times\vec{\sigma}_3)_z}{2m}\left(1+\frac{k_1^z}{m}-\frac{\vec{q}\cdot\vec{k}_1}{4m^2}\right) \\ &+ \frac{\vec{\sigma}_3\cdot(\vec{k}_3\times\vec{q})(\vec{q}\times\vec{\sigma}_1)_z}{8m^3} + \frac{(\vec{q}\times\vec{\sigma}_3)_z\vec{\sigma}_1\cdot(\vec{k}_1\times\vec{q})}{8m^3} \\ &+ i\frac{\vec{\sigma}_3\cdot(\vec{k}_3\times\vec{q})}{4m^2} - i\frac{\vec{\sigma}_1\cdot(\vec{k}_1\times\vec{q})}{4m^2} + O\left(\frac{k_1^2}{m^2}, \frac{k_3^2}{m^2}\right)\Bigg\} \end{aligned}$$

- helicity-flip interaction $\longrightarrow$ $\mathbf{f_{1T}^{\perp\mathcal{Q}}(x, k_T) \neq 0}$
- extreme NR limit $\longrightarrow$ no **"small components"** of the four-spinors $\longrightarrow$ no helicity-flip

$f_{1T}^{\perp\mathcal{Q}}(x, k_T) \neq 0$ comes from the Interference of the "small" and "large" components in the four-spinors of the free quark states

The interaction is to be calculated between proton states $\Psi_{r\,S_z=\pm1}$ in a CQM $\Rightarrow$ e.g., Isgur-Karl

Sivers function in Isgur-Karl: Higher waves decomposition



- Nucleon state: (we use the 3 first waves)

  $|N\rangle = a|^2S_{1/2}\rangle_S + b|^2S'_{1/2}\rangle_S + c|^2S_{1/2}\rangle_M$

  Notation: $|^{2S+1}X_J\rangle_t$; $t = A, M, S$ = symmetry type
  From spectroscopy:
  $a = 0.933$, $b = -0.275$, $c = -0.233$

- "Higher waves"
  - Importance of small components in the proton wave function
  - relevance of further analysis with other (relativistic) models.

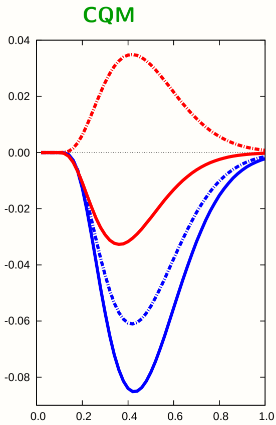
Spin dependence of the matrix element: Sivers function

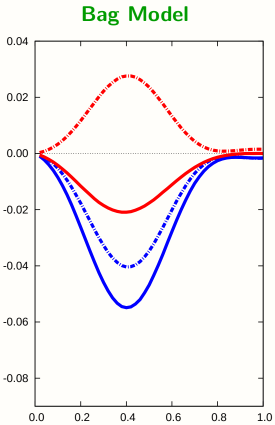
In a helicity basis, the matrix element to be evaluated is of the type

$$
\begin{aligned}
& 3\,\langle \psi(\vec{k})\,\varphi_c\,\frac{1}{\sqrt{2}}\left(\phi_{MA}\chi^{\uparrow}_{MA}+\phi_{MS}\chi^{\uparrow}_{MS}\right)|\,\frac{1\pm\tau_3(3)}{2}\,\hat{O}_{spin}(\vec{k})|\psi(\vec{k})\,\varphi_c\,\frac{1}{\sqrt{2}}\left(\phi_{MA}\chi^{\downarrow}_{MA}+\phi_{MS}\chi^{\downarrow}_{MS}\right)\rangle \\
= \quad & 3\left(-\frac{2}{3}\right)\frac{1}{2}\Big\{\phi^*_{MA}\,\frac{1\pm\tau_3(3)}{2}\,\phi_{MA}\,\langle\psi(\vec{k})\chi^{\uparrow}_{MA}|\hat{O}_{spin}(\vec{k})|\psi(\vec{k})\chi^{\downarrow}_{MA}\rangle \\
& \qquad +\phi^*_{MS}\,\frac{1\pm\tau_3(3)}{2}\,\phi_{MS}\,\langle\psi(\vec{k})\chi^{\uparrow}_{MS}|\hat{O}_{spin}(\vec{k})|\psi(\vec{k})\chi^{\downarrow}_{MS}\rangle+0+0\Big\} \\
\mathbf{u} \quad \Rightarrow \quad & 3\left(-\frac{2}{3}\right)\frac{1}{2}\Big\{1\,\langle\psi(\vec{k})\chi^{\uparrow}_{MA}|\hat{O}_{spin}(\vec{k})|\psi(\vec{k})\chi^{\downarrow}_{MA}\rangle+\frac{1}{3}\,\langle\psi(\vec{k})\chi^{\uparrow}_{MS}|\hat{O}_{spin}(\vec{k})|\psi(\vec{k})\chi^{\downarrow}_{MS}\rangle\Big\} \\
= \quad & -\left(f(\vec{k})+\frac{1}{3}g(\vec{k})\right) \\
\mathbf{d} \quad \Rightarrow \quad & 3\left(-\frac{2}{3}\right)\frac{1}{2}\Big\{0\,\langle\psi(\vec{k})\chi^{\uparrow}_{MA}|\hat{O}_{spin}(\vec{k})|\psi(\vec{k})\chi^{\downarrow}_{MA}\rangle+\frac{2}{3}\,\langle\psi(\vec{k})\chi^{\uparrow}_{MS}|\hat{O}_{spin}(\vec{k})|\psi(\vec{k})\chi^{\downarrow}_{MS}\rangle\Big\} \\
= \quad & -\left(\frac{2}{3}g(\vec{k})\right)
\end{aligned}
$$

No proportionality between the u and d-distributions due to the spin and momentum dependence of the operator!

Comparison of the T-odd functions





$\Rightarrow$ Boer-Mulders bigger than Sivers function for both flavors

$\Rightarrow$ same trend in both models for both flavors.